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A THEORETICAL MODEL FOR PREDICTING THERMAL STRATIFICATION AND SELF PRESSURIZATION OF A FLUID CONTAINER

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ABSTRACT

Thermal stratification and self-pressurization of cryogenic propellant tanks are of concern to many people in the astro-space and missile industries. A procedure, based upon the Von Karman integral method, is presented for analytically determining both the liquid thermal stratification and the extent of the associated self-pressurization. Both conical and cylindrical geometrys are treated.

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Methods for analyzing thermal stratification of cryogenic propellants have been studied by several investigators in recent years [Robbins and Rogers, 1964], [Schmidt, et al, 1961], [Schwind and Vliet, 1964], [Scott, et al, 1960], [Tatom, et al, 1964], [Tellep and Harper, 1963]. Most of these analyses have been aimed primarily at the temperature distribution within and the volume occupied by the thermally stratified layer. Some have touched on the effect of this thermal stratification on the ullage pressure [Tellep and Harper, 1963], but no intensive attack has been reported on this phase of the problem.

As a part of consultation work on the Centaur project for NASA-Lewis Research Center, the Cryogenics Division of National Bureau of Standards has undertaken an analysis which could be used with a geometry approximating that of the Centaur liquid hydrogen tank. The expressions obtained as a result of this analysis can be applied to other geometries with only slight modification of procedure. Numerical solutions of the equations involve numerical integration and/or iteration. These computations are most efficiently performed by a digital computer.

The configuration used to approximate the Centaur shape is shown in Figure 1. Since the approximate shape included a conical nose which contained liquid under some operating conditions, the basic analysis was conducted using this shape. Reduction of the resulting equation to reflect a purely cylindrical shape is readily accomplished. The approach used is based upon the classical approximate integral analysis developed by Von Karman [1946] as modified by Eckert and Jackson [1951]. Development of the equations was based upon the more detailed model shown in Figure 2.

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A free convection turbulent boundary layer was assumed with temperature and velocity variations through the layer patterned after Eckert and Jackson [1951], i. e.,.

$$\theta = \theta_w\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]$$

and

$$u = U\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\left(1-\frac{y}{\delta}\right)^4 .$$

The approach used was to equate the vertical component of the change in momentum flow through the element to the vertical forces acting on the element. An identity in x results from this procedure consisting of:

$$\left[2\pi \cos^2\gamma \frac{\partial}{\partial x}\int_0^\delta \rho u^2(R-y)\,dy\right]dx$$
$$\equiv 2\pi g\beta\int_0^\delta \rho\theta(R-y)\,dy\,dx - 2\pi R\tau_w\,dx \quad (1)$$

Substitution for the wall shear stress is made by means of the Blasius correlation [Eckert and Drake, 1959, p. 143].

$$\tau_w = 0.0228\rho U^2\left(\frac{\nu}{U\delta\cos\gamma}\right)^{\frac{1}{4}} . \quad (2)$$

Temperature differences experienced between the wall and the bulk fluid are small for the cases considered here, therefore $\rho/\rho_B \cong 1$, if the application of this assumption to arithmetical differences is avoide[d]. The arithmetic difference has been taken care of by the introduction of th[e] volumetric coefficient of expansion, β.

Evaluation of the integrals in (1) and substitution for τ_w gives

$$\cos^2\gamma \frac{\partial}{\partial x}\left[0.05232\, R U^2 \delta \left(1 - \frac{\delta}{8R}\right)\right]$$
$$\equiv \tfrac{1}{8}\, g \beta R \theta_w \delta \left(1 - \frac{4\delta}{15R}\right) - 0.0228\, R U^2 \left(\frac{\nu}{U\delta \cos\gamma}\right)^{\frac{1}{4}}. \tag{1a}$$

For the application intended here, a moderately well insulated tank is assumed, and it is found that the dominating thermal resistance is formed by the insulation so that the heat flux to the tank is essentially constant. The assumption of constant heat flux and the combination of the Blasius correlation with the Reynolds analogy [Eckert and Drake, 1959, p. 203] together with a Prandtl number correction due to Colburn [Jakob, 1949, Eckert and Drake, 1959, p. 324] results in

$$\cos^2\gamma \frac{\partial}{\partial x}\left[0.05232\, R U^2 \delta \left(1 - \frac{\delta}{8R}\right)\right]$$
$$\equiv \frac{g \beta q_w Pr^{\frac{2}{3}} \delta R}{0.1824 k\ \rho_b U} \left(\frac{U\delta \cos\gamma}{\nu}\right)^{\frac{1}{4}} \left(1 - \frac{4\delta}{15R}\right)$$
$$- 0.0228\, R U^2 \left(\frac{\nu}{U\delta \cos\gamma}\right)^{\frac{1}{4}}. \tag{1b}$$

It is now assumed that the equivalent velocity and the boundary layer thickness are of exponential form as given by

$$U = C_1 x^m$$

and

$$\delta = C_2 x^n.$$

When this substitution is made and the differentiation performed, the identity

$$0.05232\, C_1^2 C_2 (2m+n) R x^{2m+n-1} \left[1 + \frac{1}{2m+n} \frac{x}{R} \frac{\partial R}{\partial x} - \frac{C_2 (2m+2n)}{8(2m+n)} \frac{x^n}{R} \right] \cos^2 \gamma$$

$$\equiv \frac{g \beta q_w A^{\frac{2}{3}}}{0.1824 c\, \rho_B v^{\frac{1}{4}}} C_2^{\frac{5}{4}} C_1^{-\frac{3}{4}} x^{\frac{5n-3m}{4}} R \cos^{\frac{1}{4}} \gamma \left(1 - C_2 \frac{4}{15} \frac{x^n}{R} \right)$$

$$- 0.0228 \frac{v^{\frac{1}{4}}}{\cos^{\frac{1}{4}} \gamma} C_1^{\frac{7}{4}} C_2^{-\frac{1}{4}} R x^{\frac{7m-n}{4}} \tag{1c}$$

is obtained.

Comparison of this expression with that of previous investigators [Morse, 1962] who used a semi-infinite vertical flat plate model reveals that the terms involving the ratio x^n/R result from the circular shape configuration while the term incorporating $\partial R/\partial x$ enters because of the conical shape.

Since this expression is an identity with x as the variable, there must be equality of the exponents of x. When these equalities are written for either a vertical flat plate or cylindrical configuration, the values for m and n are obtainable and consistent with values of 3/7 and 5/7 respectively. When the term resulting from the conical shape is included, the answer is undefined. As a result, the expressions developed in this paper do not fully reflect the effect of the conical shape.

To obtain a value for the constants C_1 and C_2, a second equation is developed based upon a thermal energy balance. Change in thermal energy through the fluid element is equated to thermal energy input with the following identity resulting:

$$\left[2\pi c\, \rho_B \cos\gamma \frac{\partial}{\partial x}\int_0^\delta \theta u (R-y)\, dy\right] dx$$

$$\equiv 2\pi R q_w \frac{dx}{\cos\gamma} \quad . \tag{3}$$

Upon evaluating the integral, substituting for θ_W , U and δ , and performing the differentiation, this becomes

$$\frac{1.6066\, C_1^{\frac{1}{4}} C_2^{\frac{5}{4}} Pr^{\frac{2}{3}} \cos^{\frac{3}{4}}\gamma}{\nu^{\frac{1}{4}}}\, x^{\frac{m+5n}{4}} \left\{ \left[\frac{1}{R}\frac{\partial R}{\partial x} - 0.1307\, C_2\, n\, \frac{x^{n-1}}{R}\right] \right.$$

$$\left. + \left(\frac{m+5n}{4}\right)\left(\frac{1}{x}\right)\left[1 - 0.1307\, C_2 \frac{x^n}{R}\right] \right\} \equiv 1 \quad . \tag{3a}$$

Again, a check of the exponents reveals consistency except in the case of a conical shape.

Inserting values for m and n and solving (1c) and (3a) simultaneously for C_1 and C_2 yields the following expressions:

$$C_1 = \frac{[K_1 K_2]^{\frac{5}{14}}}{K_7 K_6^{\frac{1}{14}}} \left[\frac{K_4}{K_3} + Pr^{\frac{2}{3}} \frac{K_6}{K_5}\right]^{-\frac{5}{14}} \left[\frac{g\beta q_w}{c\, \rho_B}\right]^{\frac{5}{14}} \left[\frac{Pr^{\frac{4}{21}}}{\nu^{\frac{1}{14}} \cos^{\frac{11}{14}}\gamma}\right] \tag{4a}$$

and

$$C_2 = \left[\frac{1}{K_1 K_2 K_6^{11}}\right]^{\frac{1}{14}} \left[\frac{c\, \rho_B \nu^3}{g\beta q_w Pr^8 \cos^{23}\gamma}\right]^{\frac{1}{14}} \left[\frac{K_4}{K_3} + Pr^{\frac{2}{3}} \frac{K_6}{K_5}\right]^{\frac{1}{14}} , \tag{4b}$$

where

$$K_1 = 22.735$$
$$K_2 = 1 - 0.2667\, C_2 \frac{x}{R}^n$$
$$K_3 = 539.86$$
$$K_4 = 1 + \frac{x}{R}\frac{\partial R}{\partial x} - 0.1818\, C_2 \frac{x}{R}^n$$
$$K_5 = 1211.7$$
$$K_6 = 1 + \frac{x}{R}\frac{\partial R}{\partial x} - 0.2239\, C_2 \frac{x}{R}^n$$
$$K_7 = 6.6624 \; .$$

An iteration procedure will permit determination of C_2 , although a good first approximation can be obtained by assuming a vertical flat plate geometry, thus making K_2, K_4 and K_6 equal to 1.

Either of the above methods may be used to determine the constants thus providing a description of the boundary layer growth.

Using the developed expressions for the boundary layer, a growth rate for the thermally stratified layer is now determined as

$$\frac{\partial \Delta}{\partial t} = \frac{1}{A_\Delta - A_{\delta\Delta}} \left(\frac{\partial V}{\partial t}\right)_\Delta \; . \tag{5}$$

Evaluating $A_\Delta, A_{\delta\Delta}$ and $(\partial V/\partial t)_\Delta$ gives an expression for growth rate as

$$\frac{\partial \Delta}{\partial t} = \frac{0.2928\, U_\Delta \delta_\Delta \cos \gamma}{R_\Delta - \delta_\Delta\left(2 - \frac{\delta_\Delta}{R_\Delta}\right)}\left[1 - 0.1860\, \frac{\delta_\Delta}{R_\Delta}\right] . \tag{5a}$$

A numerical integration may now be used to produce a relation between Δ and t .

STRATIFIED LAYER TEMPERATURE GRADIENT

To derive an analytical expression for the temperature gradient in the thermally stratified layer, it is assumed that the boundary layer decays completely in traversing the layer. This action is mathematically expressed as

$$\delta_\ell = \delta_\Delta \left(1 - \frac{Z}{\Delta}\right)^{r}$$

and

$$U_\ell = U_\Delta \left(1 - \frac{Z}{\Delta}\right)^{p},$$

where δ_Δ and U_Δ are the boundary layer thickness and equivalent velocities at the bottom of the layer. (See Figure 3).

An identical procedure to that previously used for setting up a momentum equation is employed with the result that p and r are determined to have values of 3/7 and 5/7 respectively.

Establishment of an energy equation proceeds along identical lines but requires the addition of a term representing energy transport across the inner face of the element. After evaluating integrals, substituting for θ_W, U_ℓ and δ_ℓ, and performing any required differentiations, a differential equation in $\theta_{B\ell}$ and Z is obtained. Since it appears simpler to solve the equation by numerical integration, the equation is arranged with this in mind and is given as

$$\frac{\partial \theta_{B\ell}}{\partial Z} = \left[\frac{6.8307\, q_W}{c \rho_B U_\Delta \delta_\Delta \cos^2\gamma}\right] \left\{ \frac{1 - \left[\frac{1.6066\, Pr^{\frac{2}{3}} U_\Delta^{\frac{1}{4}} \delta_\Delta^{\frac{5}{4}} \cos^{\frac{9}{4}}\gamma}{\nu^{\frac{1}{4}} \Delta}\right] \left[0.2239 \frac{\delta_\Delta}{R}\left(1 - \frac{Z}{\Delta}\right)^{\frac{5}{7}} - 1 + \left(1 - \frac{Z}{\Delta}\right)\frac{\Delta}{R}\frac{\partial R}{\partial Z}\right]}{\left(1 - \frac{Z}{\Delta}\right)^{\frac{8}{7}} \left[1 - 0.1861 \frac{\delta_\Delta}{R}\left(1 - \frac{Z}{\Delta}\right)^{\frac{5}{7}}\right]} \right\} \quad (6)$$

The above equation enables us to define a vertical temperature pattern in the stratified layer in the absence of vaporization or condensation at the surface.

ULLAGE SPACE TEMPERATURE GRADIENT

The approach utilized above can be applied to the ullage space when modified to reflect variable wall heat flux. Experimental evidence [Maxson, 1963] shows that significant temperature increases can occur in the ullage space under pressurized conditions. As a result, the wall heat transfer rate will change due to a significant increase in the inner wall temperature. It then becomes necessary to describe wall heat flux as a function of temperature difference. This is accomplished by assuming a simple linear relation, i. e.,

$$q_W = \Omega(T_a - T_\ell) = \Omega\theta_{a\ell} .$$

Since $\theta_{a\ell} = \theta_a - \theta_{B\ell}$, we have $q_W = \Omega(\theta_a - \theta_{B\ell})$. Substitution of this term for q_W and development of the equation yields the differential equation

$$\begin{aligned}
&\frac{\partial\theta_{a\ell}}{\partial z}\left\{\left[\frac{Pr^{\frac{2}{3}}\Omega U_\Delta^{\frac{1}{4}}\delta_\Delta^{\frac{5}{4}}\cos^{\frac{1}{4}}\gamma}{0.0228\,c\rho_B\,\nu^{\frac{1}{4}}}\right]\left[0.004785\frac{\delta_\Delta}{R}\left(1-\frac{z}{\Delta}\right)^{\frac{12}{7}} - 0.03663\left(1-\frac{z}{\Delta}\right)\right]\right. \\
&\left. + U_\Delta\delta_\Delta\left[0.1464\left(1-\frac{z}{\Delta}\right)^{\frac{8}{7}} - 0.02724\frac{\delta_\Delta}{R}\left(1-\frac{z}{\Delta}\right)^{\frac{12}{7}}\right]\right\} \\
&+ (\theta_a - \theta_{B\ell})\left\{\left[\frac{Pr^{\frac{2}{3}}\Omega U_\Delta^{\frac{1}{4}}\delta_\Delta^{\frac{5}{4}}\cos^{\frac{1}{4}}\gamma}{0.0228\,c\rho_B\,\nu^{\frac{1}{4}}\Delta}\right]\left[0.03663\frac{\Delta}{R}\left(1-\frac{z}{\Delta}\right)\frac{\partial R}{\partial z} - 0.03663\right.\right. \qquad (7) \\
&\left.\left. + 0.004785\left(\frac{12}{7}\right)\frac{\delta_\Delta}{R}\left(1-\frac{z}{\Delta}\right)^{\frac{5}{7}}\right] - \frac{\Omega}{c\rho_B\cos^2\gamma}\right\} \equiv 0 .
\end{aligned}$$

Solution of this equation requires a combined iteration and numer: integration procedure.

PRESSURIZATION COMPUTATIONS

In order to use the above developed equations to predict pressurization rates, it is necessary to couple the results for the ullage space and the liquid phase. To accomplish this we look at a differential volume element described by

$$dV = \pi R^2 d\psi .$$

Note that the variable ψ introduced here is a variable over the entire height of the ullage space regardless of whether the ullage stratified layer has occupied the entire ullage space, (Figure 1). The mass contained in this element is then given by

$$dM = \pi \rho R^2 d\psi ,$$

and the total mass by

$$M = \pi \int_0^h \rho R^2 d\psi . \tag{8}$$

The fluid mass density, ρ , is a function of the local temperature and pressure, while R is a function of ψ .

Since equation (7) will provide information on θ_{BL} as a function of Z , and therefore of ψ , the total mass may be determined from

$$M = \pi \int_0^h \rho(\theta_{BL}, P_S)\left[R_S - \psi \tan\gamma\right]^2 d\psi , \tag{8a}$$

where ψ includes the region below the stratified vapor layer as well as the layer where a thermal gradient exists. A combined iteration and numerical integration procedure must be used. A pressure is assumed and the expression numerically integrated until the computed mass compares with the known mass from initial conditions or a previous step. The pressure obtained from this procedure has a matching saturation temperature which is compared with the liquid surface temperature determined earlier from the thermal analysis of the stratified liquid layer. If a satisfactory match is not made, then vaporization of liquid or condensation of gas is permitted to bring the liquid surface temperature into equilibrium with the computed pressure. The different temperature patterns which result from liquid temperature greater than saturation temperature and liquid temperature less than saturation temperature are shown in Figure (4a) and (4b) respectively. The mass vaporized is added to the initial ullage mass (determined by assuming saturated gas to fill the ullage at the beginning of pressurization) and the new mass used in the next step. Computation of the mass vaporized, DM_v, is accomplishec by use of the equation

$$DM_V = \frac{\pi}{L_V} \int_0^{z_1} c \rho_B (\theta_l - \theta_s) R^2 dz_1 \quad , \tag{9}$$

where z_1 is measured from the liquid surface downward and L_v represents the latent heat of vaporization.

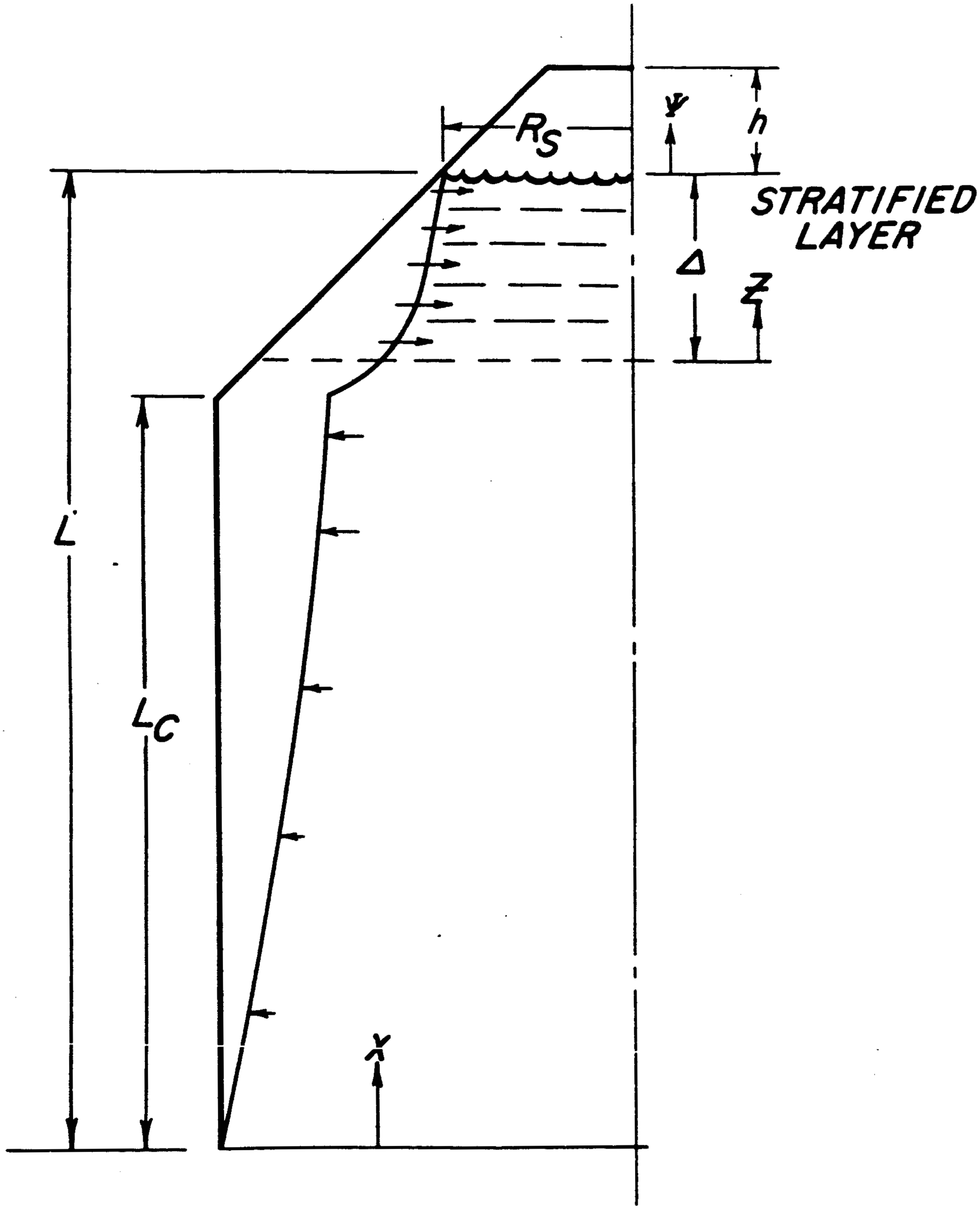

Figure 1. Assumed Geometry

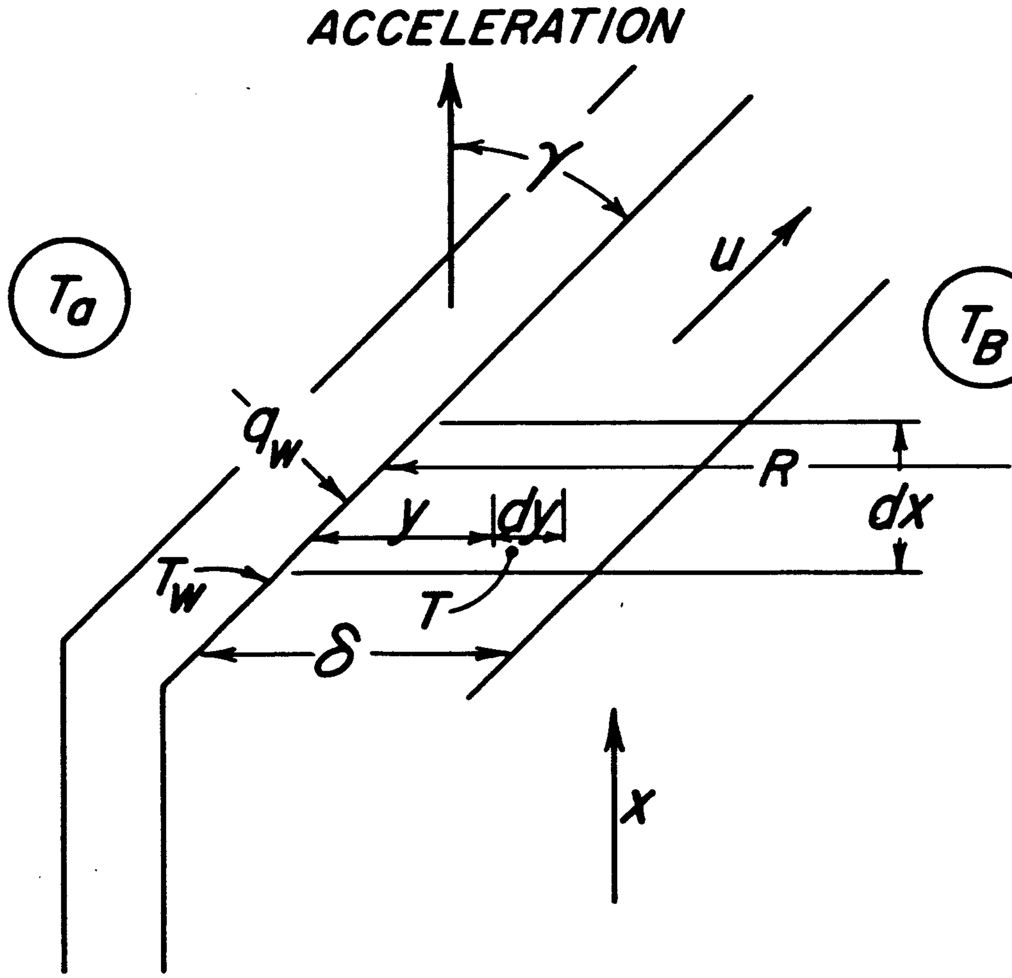

Figure 2. Element Considered For Boundary Layer Growth

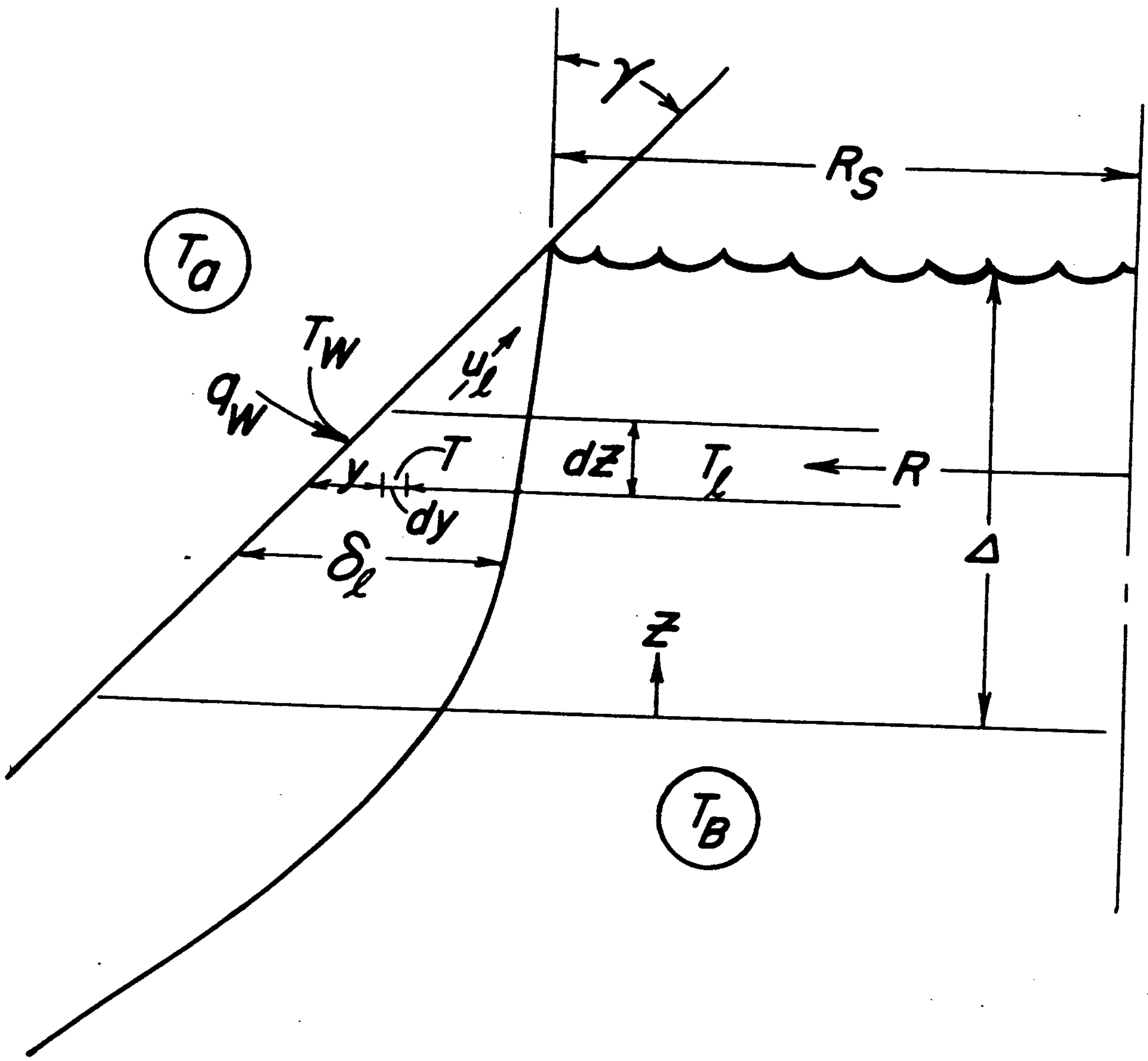

Figure 3. Element Considered For Boundary Layer Decay

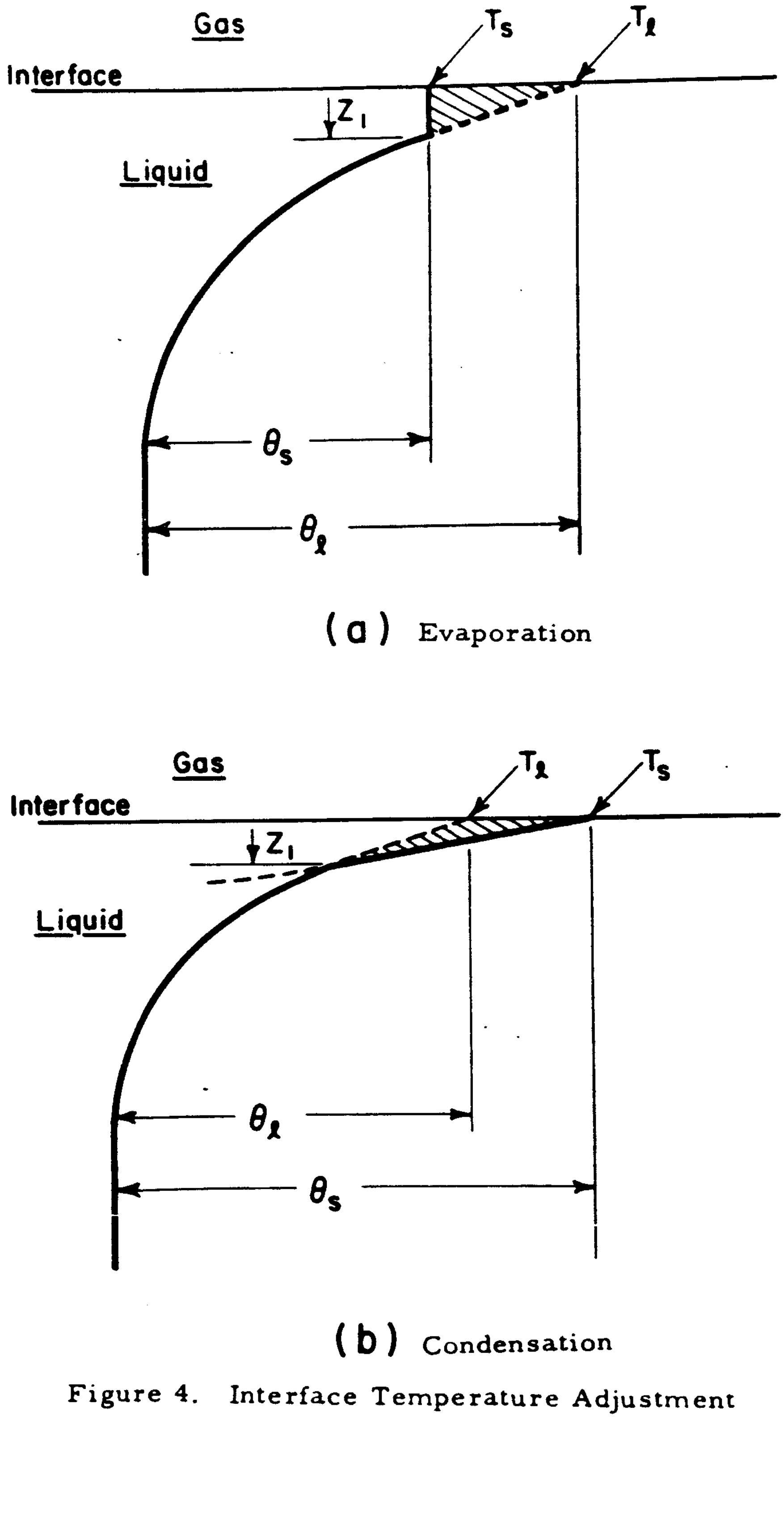

Figure 4. Interface Temperature Adjustment

Nomenclature

A - cross section area

C_1, C_2, m, n, r, p - derived constants

c - specific heat

$\mathcal{G}_{RL}$ - modified Grashof number,

g - acceleration

h - vapor space height

K - substitutional term

k - thermal conductivity

L - liquid depth

L_c - length of cylindrical tank

M - mass

Pr - Prandtl number

q - unit area heat flux

R - tank radius

R_s - radius at the liquid surface

T - absolute temperature

t - time

u - local velocity in boundary layer

U - equivalent free stream velocity

V - volume

x - vertical distance in tank

y - horizontal distance from tank wall

z - vertical distance from bottom of stratified layer

β - volumetric coefficient of thermal expansion

γ - half angle of conical nose

Δ - stratified layer thickness

δ - boundary layer thickness

θ - temperature difference, $T - T_B$.

$\theta_a = T_a - T_B$

$\theta_{al} = T_a - T_l$

$\theta_{Bl} = T_l - T_B$

$\theta_W = T_W - T_B$

μ - dynamic viscosity

ν - kinematic viscosity

ρ - fluid mass density

τ - viscous shear stress

ψ - vertical distance in ullage space

Ω - overall tank wall heat transfer coefficient

<u>Subscripts</u>

a - ambient conditions (external to tank)

B - at bulk fluid condition

l - in stratified layer

n - in the conical nose

W - at tank wall

S - in the ullage space

Δ - at bottom of stratified layer

δ - pertaining to boundary layer

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